Optimal Multiproduct and Multiechelon Supply Chain Network Design

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Abstract. This work proposes a novel approach for the optimal design of multiproduct supply chain networks (*SCN*). Through a mixed integer linear programming (*MILP*) formulation the aim is to establish the structure of facilities that minimizes costs over the planning horizon, taking into account all the *SCN*'s distinctive characteristics. We develop a generalized approach optimally determining the location of various types of facilities, multiproduct flows and demand fulfillment from any node in the network. This allows capturing the intertwined nature of decisions, leading to more efficient results. The proposed approach does not limit the number of echelons or layers. Instead, through a novel formulation, the optimal number of echelons is determined by the model, depending on the product to be supplied. To capture the economies of scale governing capital investments and operational costs, different types of facilities are proposed. Besides, the transportation expenses take different unitary costs according to the type of nodes being connected. Finally, the concept of waiting cost is introduced in order to capture the responsiveness of the *SCN* through the measurement of the time required to fulfill the clients' demands. A case study with different demand patterns and data structures is addressed to assess the potentials and efficiency of the *SCN* designs obtained with the proposed approach.

Keywords: supply chain design, optimization, multiproduct

1 Introduction

The optimal design of supply chain networks (*SCN*) has become a strategic field in recent years because of the implication of the logistic expenses in the overall cost of organizations. With some differences according to the sector, logistic costs average 7% to 9% of the sales of a company, reaching up to 30% for certain chemistry industries. In global terms, the *IMF* roughly estimates that the logistic costs are about 12% of the global GDP [1]. As a result, the optimization of the supply chain network design and operation may have an enormous impact on the total costs.

A supply chain network is typically represented by complex graphs including flows of goods, materials and information linking the different nodes making part of it. These nodes stand for suppliers, manufacturing plants, warehouses, distribution centers, cross-docking facilities and/or demand points. Nowadays, market globalization along with shortening of product life-cycles and the need for high standards of responsiveness challenge modern supply chains to be agile and flexible enough to face a changing environment [2]. Thus, the search for efficient, flexible and robust network designs gives rise to a very interesting problem. Researchers have addressed this topic with different tools, including heuristics, optimization and simulation models, though several issues still remain open [3].

The design of a *SCN* is a strategic and long-term problem to be addressed by one or a group of organizations (extended supply chain network concept). It involves decisions on the location and type of facilities to be built, suppliers selection, products to store, inventory policies and transportation modes. The aim is to fulfill customer demands while minimizing the net present value of capital expenditures and operational costs. Furthermore, the network responsiveness should be planned accounting for the service level required by the customers. It is relevant to point out that the supply chain network design (*SCND*) involves decisions requiring major capital investments in infrastructure, material handling equipment and management systems, typically facing a long-term payback period. The resulting *SCND* does not usually allow for substantial changes and re-designs, thus being critical for an optimal supply chain operation (*SCO*). As a result, the development of a new *SCND*, with robustness and sensibility considerations, has become a very important issue. Several works in the related literature address the *SCND* problem through different approaches and conceptual models trying to capture all the distinctive problem characteristics. Network design decisions impose hard constraints to the *SCO*, restricting the medium-term and short-term decisions to be made. Thus, recent research has addressed the problem in an integrated manner, deciding on the *SCN* design and operations at the same time [3].

One of the main weaknesses detected in the literature is the absence of generalized models addressing current issues related to facility sizing and location in multiproduct supply chain network problems. The vast majority of the works have focused on Fixed-Supply Chain Network Design (*F-SCND*) approaches, which mainly predetermine the number of layers or "echelons" in the *SCN* together with the type of facility to be installed in each one (typically factories, warehouses and/or distribution centers). In fact, according to Melo et al. [4], about 80% of the works assume networks with only one or two echelons, and an important number of them account for the distribution of a single product. Moreover, Farahani et al. [7], updates previous reviews and states that 43 of 50 recent works study *SCND* problems considering only two and three echelons. Also, little attention has been paid to the inner logistic flows and the possibility of direct supplies from not-end echelons to final clients. In conclusion, *F-SCND* approaches are based on rigid frameworks and assumptions that usually lead to suboptimal solutions. To overcome these weaknesses, Generalized-Supply Chain Network Design (*G-SCND*) models have been recently proposed with the aim of representing more flexible networks, deciding on the location of various types of facilities in several nodes, multiproduct flows, non-hierarchical relationships between facilities and demand fulfillment from any node in the network, among other features [5][6]. The potential of the *G-SCND* approaches rely on the ability to tackle operational issues and solve trade-offs along the structure, capturing the intertwined nature of decisions and leading to more efficient results.

Figure 1 shows a graphical comparison between classical *F-SCND* and modern *G-SCND* models, where *MP* stands for *Manufacturing Plant*, *SF* for *Storage Facility*, and *DP* means *Demand Point*.

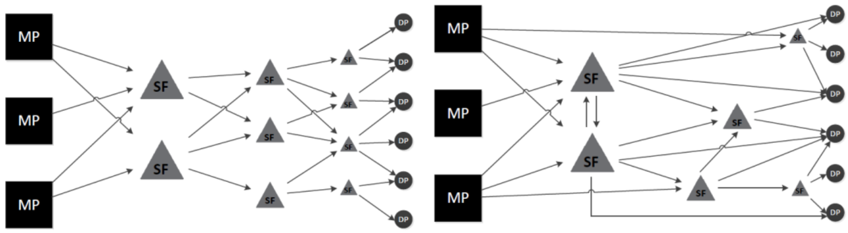

Fig. 1. The classical F-SCND model (left) against a modern G-SCND structure (right).

This paper proposes a novel approach for the optimization of *G-SCND* problems including all components commonly found in modern supply chains, yielding a comprehensive conceptual model. In contrast to previous contributions, the new formulation comprises multiproduct flows and unlimited number of echelons or layers, accounting for both operational costs and capital investments. The novel concepts introduced are the elimination of pre-determined echelons' structures and the capture of economies of scale through the modeling of a set of facility types that are able to be installed in any potential location. The main difference between them is given in terms of the minimum and maximum flows to be handled. Fixed costs, unitary handling costs and transportation costs are also dependent on the type of facility involved. Finally, the concept of waiting cost is introduced in order to assess the responsiveness of the *SCN*, by accounting for the time required to serve the clients.

2 Problem Statement

This work addresses a generalized supply chain network design problem with the main objective of determining the optimal location for a group of storage facilities (*SF*) with different sizes, in order to supply several demand points (*DP*) with various product families (*PF*) produced by preexisting manufacturing plants (*MP*) over a long-term planning horizon. The locations selected by the optimization model will configure a *SCN* with a non-predetermined number of echelons minimizing the overall net present costs. A typical *SCN* involves the management of several stockeable units. In order to reduce the model size, this work aggregates products in families. This is usually made in a previous phase considering intrinsic properties of each product such as density, value, size, and handling difficulty. Finally, we assume that each *DP* has its own annual and deterministic demand pattern that has to be fulfilled by the network.

In this problem, the storage facilities to be installed, among a determined set of potential nodes, are of three types: large, medium or small. In other words, each potential node, if selected, has to also adopt one of these types of facilities. In addition, all the investments in new *SFs* are placed at the beginning of the time horizon. The differences between facility types aim to capture the economies of scale governing both capital investment and operational cost when the size of the storage facility and the quantities being handled vary. In fact, economies of scale functions are used to model capital expenditures in new infrastructure and operational costs. This typically non-linear relationship between the size of a facility and the corresponding cost is such that every additional unit size added to a facility is more economic than the previous one. A similar behavior is observed for the unit handling and transportation costs. To model the economies of scale, it is considered a minimum investment and an additional fixed cost for each *PF* being allocated, according to the type of facility. Likewise, annual fixed costs are considered for each type of *SF* selected and *PF* allocated to it. Other main parameters that differentiate each type of location, in addition to capacity limitations, are *PF*-dependent lower and upper bounds on the product flow. Furthermore, global minimum and maximum bounds are imposed for each type of facility (small, medium, large). These bounds are in close relation with the facility size. On the other hand, unit operational handling costs tend to be smaller for larger facility types due to the use of more efficient equipment. This is captured by defining different unit handling costs for each *PF* at each type of facility.

A series of existing manufacturing plants (*MP*) are the origin of product supplies. Each *MP* has its own capacity (annual availability) of *PFs*, being a decision variable of the proposed model the selection and definition of quantities to be purchased. This work does not address the location of production facilities. The products, once acquired, move across certain nodes of the *SCN* until being finally transported to the demand points. The number of movements a *PF* makes before being delivered to a *DP* has no constraints. Also, it is important to observe that no single sourcing constraints are imposed in the formulation, allowing for multiple-sourcing supply to a specific *DP*. Moreover, there are no limitations in the type of facility to finally supply a *DP*. It is also important to underline the fact that each location shares its infrastructure to manage a set of *PFs*. This should be encouraged by the model because of the aggregation of operations to reduce the impact of investment and fixed costs.

With regards to the transportation costs, a fixed cost is paid for each pair of nodes being linked, meaning an annually fixed amount paid for each open route. Also, a variable transportation cost is computed depending on the *SF* type being linked and the *PF* to transport. This intends to capture the economies of scale relating transportation costs and load sizes.

Besides the size, bounds on flows, transportation conditions, fixed and variable costs, each type of storage facility must have a stock policy. Although some authors prefer to not consider this aspect at the *SCND* phase, it could imply hard constrains to the further supply chain operation. Even though this work does not address the selection of inventory policies, a specific stock policy is pre-defined for each type of facility and *PF* allocated to it. This is made following a basic empiric rule stating that large facilities have larger flows and lead to larger average stock levels. Finally, the stock policy definition yields the inventory holding costs incurred by the system.

Another novel feature included is the waiting cost, which is meant to assess the responsiveness and service level of the *SCN*. This is computed from the total time needed to provide a *DP* with a certain *PF*. The unit waiting cost is related with the impact of unsatisfied demand and the importance of a rapid delivery when *DPs* require products to their normal operation. The total time required to transport a certain item is the sum of travel and management times. The latter accounts for the activities to prepare the load before the final supplying. The management time directly depends on the type of facility, being larger for large locations and more expeditious for small ones. Waiting costs are very significant because large unit waiting costs would lead to extended *SCN*, with several facilities near *DPs*, while low unit waiting costs would lead to more compact *SCNs*.

The proposed *SCN* conceptual model does not limit the number of echelons or layers. Instead, through a novel formulation, the optimal number of echelons is determined by the model, depending on the type of product to be supplied. The resultant network could be composed of just a few facilities in a two-echelon structure (a minimum of two echelon is imposed) or, in the opposite, many facilities could be combined in a multi-echelon complex scheme. The final solution determines a *sub-SCN* design for each product family within a general *SCN* that is shared for all the *PF*, gaining the benefit of consolidation and economies of scale. Also, it is important to highlight the deterministic nature of the data, omitting uncertainty related to security stocks, demand rates and transportation lead-times.

The decisions to be made by the *G-SCND* optimization model seek to determine: (1) number of facilities to install, (2) type of facilities, (3) *PF* allocation to facilities, and (4) annual flows between nodes (*MP* – *SF* – *DP*). Summarizing, the main concept introduced is the flexible relationship between the different nodes in the network, allowing for the total or partial fulfillment of the demand from any facility. Products are not forced to move across the whole *SCN* structure, and the model must decide on the most convenient *sub-SC* configured for each *PF*. Furthermore, it is allowed the transportation in any direction of the route linking a pair of nodes. The optimization model solves numerous trade-offs to finally adopt the best design, minimizing the net present value of the overall costs over the planning horizon.

3 Mathematical Formulation

A Mixed Integer Linear Programming (*MILP*) formulation is proposed in order to mathematically represent the problem described in the previous section. The aim is to supply a set of demand points $J=\{1, 2, ..., j\}$ with a group of product families $F=\{1, 2, ..., f\}$ over a planning horizon $T=\{1, 2, ..., \tau\}$. Each *DP* has a given geographical position and a specific annual demand $D_{fj\tau}$ (tons) of f for year τ. Products can be purchased in a set $I=\{1, 2, ..., i\}$ of manufacturing plants (*MP*) with a given geographical position. Besides, each *MP* has a maximum annual availability av_{fi} and a purchase cost cp_{fi} (U$S/ton). Usually, suppliers include in their costs the transportation to destination, absorbing the logistic costs in the pricing agreement. In this model, moreover, it is assumed that more efficient solutions can be achieved by the individualization of each component of the cost, particularly differentiating purchase and logistic terms. To fulfill *DP* demands, a network of storage facilities (*SF*) has to be installed to supply the products from *MPs*. A set $K=\{1, 2, ..., k\}$ of potential *SF* nodes, with their corresponding positions is proposed for the design of the *SCN*. It is important to mention that the discrete spatial approach stems from the need to avoid bi-linear terms in the formulation. If a continuous spatial approach was adopted, nonlinearities would appear because of the consequent variable nature of the distance. In our model, if a node *k* is adopted as a *SF*, the binary variable v_k takes value one, and zero otherwise. Then, if the selected node is decided to be of type *t* (*Large-Medium-Small*) we force the binary variable w_{kt} to be equal to one (Eq. 1). In short, if the model decides to install a *SF* in node *k*, it has also to be characterized. Moreover, when a node *k* is selected, any *PF f* can be allocated to it. The binary variable u_{fk} takes value one if *f* is allocated to *k* and zero otherwise (Eq. 1).

$$\sum_{t} w_{kt} = v_k \quad \forall k \quad ; \quad u_{fk} \leq v_k \qquad \forall f, k \tag{1}$$

To simplify the model, three multidimensional sets are introduced. *FJ(f,j)* comprises the *PF* required by each *DP*, avoiding families not demanded by certain customers. In turn, *FT(f,t)* includes families *f* that are able to be allocated to each type of facility *t*. In many situations, certain items cannot be allocated to all types of facilities. For instance, in the oil and gas industry, equipment and materials required for new locations development is only stored in large facilities due to its storage cost. In general terms, if the facilities are small and designed for an expeditious response with low management times and small loads, certain big items are excluded. The last multidimensional set is *TK(t,k)*, which includes the types *t* of *SF* allowed to be installed in a potential node *k*, accounting for geographical issues and/or others limitations.

The final capacity of a *SF* is related to the number of products being handled and the maximum inventory expected for all of them. It is assumed that the allocation of a *PF* to a certain type of *SF* forces to handle at least a minimum amount (q_{ft}^{lo}) every year. Additionally, it is imposed a maximum annual flow (q_{ft}^{up}). Usually, this is not restrictive due to the reduction of operational cost as the size of the type of facilities increases. Similarly, global constraints are imposed to the minimum (tq_t^{lo}) and maximum (tq_t^{up}) flows that justify the installation of a *SF* of type *t*.

In order to fulfill the demand of every *PF* at every *DP*, the positive variable $QFC_{fkj\tau}$ accounts for the annual flow of *f* from *k* to *j*. The total flow towards a *DP* has to be equal to the corresponding demand (Eq. 2). No limitations are imposed on the type *t* of *SF* serving customers, allowing the model to select larger or smaller locations, near or far away. Also, note the multi-sourcing possibility, and that it is not allowed to deliver a *PF* directly from a *MP*, meaning that every *DP* must be supplied just from *SFs*. This condition imposes a minimum of two movements (echelons) to reach the *DPs* from *MPs*.

$$D_{fj\tau} = \sum_{k} QFC_{fkj\tau} \qquad \forall f, j, \tau \in FJ(f,j) \tag{2}$$

Other flows characterize the movements within the *SCN*. Basically, the primary and inner logistics, meaning the flows from *MPs* to *SFs* and the flows between a pair of *SFs*, respectively. The variable $QFM_{fik\tau}$ represents the annual amount of *f* purchased to *i* and shipped to *k*. Similarly, $QFK_{fk'k\tau}$ is the flow of *f* from *k'* to *k* during year τ. Eq. 3 computes the annual flow $QTF_{fk\tau}$ of *f* moving across *k*, and the annual global flow $TQ_{k\tau}$. Furthermore, it is critical to ensure the mass-balance (Eq. 4) for each active node, guaranteeing that the annual incoming flow in a certain *SF* is equal to the annual outgoing flow, either to other *SFs* or to *DPs*.

$$QTF_{fk\tau} = \sum_{i} QFM_{fik\tau} + \sum_{k' \neq k} QFK_{fk'k\tau} \quad \forall\, f, k, \tau \;\; ; \quad TQ_{k\tau} = \sum_{f} QTF_{fk\tau} \quad \forall\, k, \tau \tag{3}$$

$$\sum_{i} QFM_{fik\tau} + \sum_{k' \neq k} QFK_{fk'k\tau} = \sum_{k' \neq k} QFK_{fkk'\tau} + \sum_{j\,/FJ(f,j)} QFC_{fkj\tau} \quad \forall\, k, f, \tau \tag{4}$$

Note that the *SCN* is assumed to operate under steady-state conditions, without stock accumulation between subsequent periods. Additionally, there is no constraint to the magnitude of the inner flows. As mentioned before, Eq. 5 establishes effective bounds for flows according to the type of *SF*, and Eq. 6 determines the corresponding overall bounds. Binary variable wq_{fkt} takes value one when w_{kt} and u_{fk} are both active, and zero otherwise (Eq. 7).

$$\sum_{t\,/FT(f,t)} q_{ft}^{lo}\, wq_{fkt} \;\leq QTF_{fk\tau} \leq \sum_{t\,/FT(f,t)} q_{ft}^{up}\, wq_{fkt} \quad \forall\, k\,, f, \tau \tag{5}$$

$$\sum_{t} tq_{t}^{lo}\; w_{kt} \;\leq TQ_{k\tau} \leq \sum_{t} tq_{t}^{up} w_{kt} \quad \forall\, k\,, f, \tau \tag{6}$$

$$wq_{fkt} \leq\; w_{kt} \;\; ; \; wq_{fkt} \leq\; u_{fk} \;\; ; \; wq_{fkt} \geq\; u_{fk} + w_{kt} - 1 \quad \forall\, f\,, k\,, t \tag{7}$$

It is assumed that bounds on flows for the different types of *SF* have a close relation with their size. However, it is necessary to make clear that flow does not mean capacity. A certain flow can be managed by two types of *SF*, each one featuring different handling costs and stock rotation indexes.

With the aim of determining the links between different types of nodes, new binary variables are used: $x_{fik\tau}$ takes value one if *MP i* supplies *SF k* with *PF f* during year τ, $y_{fkk'\tau}$ equals one if *SF k* supplies *SF k'* with *PF f*, and $z_{fkj\tau}$ is equal to one if *SF k* supplies *j* with *f* during τ. These binary variables are related between themselves through the set of constrains 8 to 11, and determine the value of other positive variables (Eq. 12 to 14). In all cases, *M* is a large enough positive number. We also restrict the availability of each *PF* at every *MP* (Eq. 15).

$$x_{fik\tau} \leq v_k \quad ; \quad x_{fik\tau} \leq\; u_{fk} \qquad \forall i, f, k, \tau \tag{8}$$

$$y_{fkk'\tau} \leq v_k \quad ; \quad y_{fkk'\tau} \leq v_{k'} \qquad \forall\, k, k', f, \tau \tag{9}$$

$$z_{fkj\tau} \leq v_k \qquad ; \qquad z_{fkj\tau} \leq\; u_{fk} \qquad \forall f, k, j, \tau\; \in FJ(f, j) \tag{10}$$

$$y_{fkk'\tau} \leq u_{fk} \;\; ; \;\; y_{fkk'\tau} \leq u_{fk'} \qquad \forall\, k, k', f, \tau \tag{11}$$

$$QFM_{fik\tau} \leq M.v_k \;\; ; \;\; QFM_{fik\tau} \leq av_{fi}.x_{fik\tau} \qquad \forall i, f, k, \tau \tag{12}$$

$$QFC_{fkj\tau} \leq D_{fj\tau}.z_{fkj\tau} \quad \forall f, k, j, \tau \tag{13}$$

$$QFK_{fkk'\tau} \leq M.y_{fkk'\tau} \quad \forall f, k, k', \tau \tag{14}$$

$$\sum_{k} QFM_{fik\tau} \leq av_{fi} \quad \forall f, i, \tau \tag{15}$$

Economic Objective Function

One of the most critical points to address in the integral study of a *SCN* is the inclusion of operational and capital investment costs in the most accurate way possible. As it was mentioned previously, the installation of *SFs* is considered at the beginning of the planning horizon. The capital investment required for a *SF* to be installed is divided into two parts. The first component is the fix capital expenditure Inv_t needed to build a *SF* of type *t.* This formulation assumes that the size of a *SF* is determined by the capacity assigned to every product family according to the associated inventory policy. By assigning a *PF* to a *SF* it is assumed that predetermined rules, such as the maximum inventory level, are adopted, which determines the overall space assigned to it. Generally, if a *PF* is allocated to a *large SF*, large quantities are assumed to be managed and large inventories are held (following the economic order quantity reasoning). The second investment component is determined by the parameter $FInv_{ft}$. It represents the additional capital (equipment and infrastructure) needed to allocate *PF f* to a *SF* of type *t*. This parameter includes intrinsic properties like specific volume and storage requirements. Observe that we do not consider the effect of sharing specific equipment between families. The capital expenditure in *SF k*, if adopted, is then:

$$Capex_k = \sum_{t} w_{kt}.Inv_t^b + \sum_{f}\sum_{t} FFInv_{fkt} \tag{16}$$

$$FFInv_{fkt} \geq FInv_{ft}.wq_{fkt} \quad \forall f, k, t \tag{17}$$

Where the positive variable $FFInv_{fkt}$ represents the specific investment associated to the allocation of family *f* to facility *k* of type *t*. The total capital investment *TI* in the network and the annual Total Purchase Cost TPC_τ is computed by Eq. 18.

$$TI = \sum_{k} Capex_k \quad ; \quad TPC_\tau = \sum_{i}\sum_{k}\sum_{f} QFM_{fik\tau}.cp_{fi} \qquad \forall \tau \tag{18}$$

A fixed annual maintenance cost $ffmc_f$ is charged if a *PF* is decided to be handled and stored in a *SF*. It is economically beneficial, in administrative terms, to have the products of the same family pooled in one *SF*, thus penalizing the splitting. The parameter $ffmc_f$is considered independent of the type of *SF* adopted. Additionally, each type of *SF* must pay a fixed cost fmc_t due to the associated administrative expenditures. Maintenance and administrative annual fixed costs ($Tffmc_\tau$ and $Tfmc_\tau$) are:

$$Tffmc_\tau = \sum_{f}\sum_{k} ffmc_f.u_{fk} \quad ; \quad Tfmc_\tau = \sum_{k}\sum_{t} fmc_t.w_{kt} \quad \forall \tau \tag{19}$$

This formulation assumes unit operational handling costs dependent on the facility size. The related economy of scale is captured considering an unit operational cost that is larger as the size of the facility is reduced. The unit handling cost vhc_{ft} is incurred for handling one ton of *PF f* in a *SF* of type *t*. Then, the total annual operational cost (TOC_τ) is:

$$TOC_\tau = \sum_{k} Opex_{k\tau} \quad ; \quad Opex_{k\tau} = \sum_{f}\sum_{t} QTTF_{fkt\tau}.vhc_{ft} \quad \forall k, \tau \tag{20}$$

$$\sum_{t} QTTF_{fkt\tau} = QTF_{fk\tau} \;\; ; \;\; QTTF_{fkt\tau} \leq q_{ft}^{up}.w_{kt} \quad \forall f,k,t,\tau \tag{21}$$

Transportation costs are composed of a fix cost component and a variable term (Eqs. 22 to 25). Fixed transportation costs are incurred if two nodes are linked (route opening). The route opening cost is independent of the *PF*, but depends on the kind of nodes being linked. $ftc1_t$ is the fixed transportation cost between any *MP* and a *SF* of type *t*; $ftc2_{tt'}$ represents the fixed cost for a route between potential *SFs* of type *t* and *t'*; and $ftc3_t$ is the fixed cost paid for a route from a *SF* of type *t* to any *DP*. In general terms, *ftc* follows an inverse relation with the size of the *SF* being linked. In turn, the variable transportation cost component (U$S/km.ton) depends on the type of nodes being linked, also capturing the economies of scale. We assume that unit shipping costs depend on the size of the facilities that are linked. Then, $vtc1_{ft}$ is the variable transportation cost for hauling a unit of *PF f* between any *MP* and a *SF* of type *t*; $vtc2f_{tt'}$ is the unit shipping cost for moving *f* from a *SF* of type *t* to another of type *t'*; and lastly $vtc3_f$ is the unit cost to deliver *PF f* to any *DP*. Note that the last unit logistic cost does not depend on the type of *SF* because the load sizes to serve *DPs* are unique, avoiding the existence of economies of scale. Moreover, *vtc3* is usually larger than *vtc2* and *vtc1* due to the smaller size of the customer orders comparing with the other haulings. Then, the total annual transportation cost TTC_τ is:

$$TTC_\tau = TTC_{1_\tau} + TTC_{2_\tau} + TTC_{3_\tau} \tag{22}$$

$$TTC_{1_\tau} = \sum_{i}\sum_{k}\left(Tftc1_{ik\tau} + \sum_{f}\sum_{t} Dist1_{ik}.QFFM_{fikt\tau}.vtc1_{ft}\right) \tag{23}$$

$$TTC_{2_\tau} = \sum_{k}\sum_{k'}\left(Tftc2_{kk'\tau} + \sum_{f}\sum_{t'}\sum_{t} Dist2_{kk'}.QFFFK_{fkk'tt'\tau}.vtc2_{ftt'}\right) \tag{24}$$

$$TTC_{3_\tau} = \sum_{k}\sum_{j}\left(Tftc3_{kj\tau} + \sum_{f} Dist3_{kj}.QFC_{fkj\tau}.vtc3_f\right) \tag{25}$$

The distances between nodes ($Dist1_{ik}$, $Dist2_{kk'}$, $Dist3_{kj}$) in the model are computed by the euclidean norm, introducing a tortuosity factor (equal to 1.15 for the examples presented in the next section) to reflect the inexistence of perfect road grids. The positive variables $Tftc1_{ik\tau}$, $Tftc2_{kk'\tau}$, $Tftc3_{kj\tau}$, $QFFM_{fikt\tau}$, $QFFFM_{fkk'tt'\tau}$ are introduced in order to avoid nonlinearities in the relation between the binary and positive variables. We use similar formulations as the one shown in Eq. 21.

Beyond the determination of annual flows across every *SF*, it is necessary to compute the inventory levels in the system with the objective of quantifying the stock holding costs. This approach considers deterministic demand and lead times, without uncertainty sources. Then, the inventory levels of every *PF* in a *SF* of type *t* are not determined by the annual flow $QTF_{fk\tau}$ but by the inventory policy adopted. The parameter SQ_{ft} is the order size for *PF f* in a *SF* of type *t*. The *PF* average stock across the network (TAS_f) and the total annual inventory holding cost (TSC_τ) are presented in Eq. 26 considering IC_f (U$S/ton.year) as the unit inventory holding cost.

$$TAS_f = \sum_{k}\sum_{t}\frac{SQ_{ft}}{2}.wq_{fkt} \;\; \forall f \quad ; \quad TSC_\tau = \sum_{f} TAS_f.IC_f \quad \forall \tau \tag{26}$$

The flow time across the *SCN* is composed of (1) the processing time, and (2) the total transportation time. The responsiveness of the *SCN* is assessed through the waiting cost, as a measure of the service level. Given that uncertainty sources are omitted, the service level is here related to how fast the network can fulfill the *DP* demands. As the distances and the mean velocity of trucks are assumed to be known, then the lead time to link two nodes (LT_{kj}) can be readily obtained. The processing time for a single order of family f in a facility of type t (PTK_{ft}) is known. We assume that for every unit time the network takes to supply a ton of *PF*, it has to be paid a determined amount uWC_f as the so-called waiting cost. Eqs. 27 and 28 accounts for the total waiting cost TWC_τ:

$$OWC_{fkt\tau} = \sum_j QFFC_{fkjt\tau}\left(PTK_{ft} + LT_{kj}\right) uWC_f \quad \forall f,k,t,\tau \tag{27}$$

$$\sum_t QFFC_{fkjt\tau} = QFC_{fkj\tau} \;;\; QFFC_{fkjt\tau} \le M.w_{kt} \;\; \forall f,k,t,\tau \;;\; TWC_\tau = \sum_f \sum_k OWC_{fk\tau} \tag{28}$$

The *SCN* must be designed with the aim of minimizing the net present value of the overall costs (*NPC*, Eq. 29), involving all the discounted capital investment and operational costs (commonly in USD) during the planning horizon (typically 10 years). In our case study, the annual discount rate is equal to 13%.

$$Min \quad NPC = TI + \sum_{\tau=1}^{10} \frac{(TPC_\tau + TTC_\tau + Tfmc_\tau + Tffmc_\tau + TSC_\tau + TOC_\tau + TWC_\tau)}{(1+r)^{\tau-1}} \tag{29}$$

4 Results and Discussion

An illustrative case study is addressed, featuring different demand patterns, geographical distributions and unit costs, to assess the potentials of the proposed approach. It is expected to obtain diverse *SCN* designs with different types of facilities installed at different locations, and associated flow patterns, in response to the variations in the data structure. The presented example assumes that the demand is uniform for each *DP* and *PF* over the planning horizon. In consequence, the annual costs found by the model are the same for every year. Fig. 2 shows the nodes geographical distribution. Three *PFs*, two *MPs* (squares), six *DPs* (circles) and five new potential locations for *SFs* (triangles) are proposed. Besides, the location of *MPs* and *DPs* are also potential nodes to *SFs*. In this particular case, it is also assumed that *MPs* can only supply products to large and medium-size storage facilities. Small-size *SF* can only be supplied by other *SFs* with larger size. Simple variations in the data structure lead to the creation of a group of scenarios based on the same case study. The formulation was coded in GAMS 24.7 and solved using CPLEX 12.6 on an Intel Xeon X5650 with 2.67 GHz CPU and 24GB RAM.

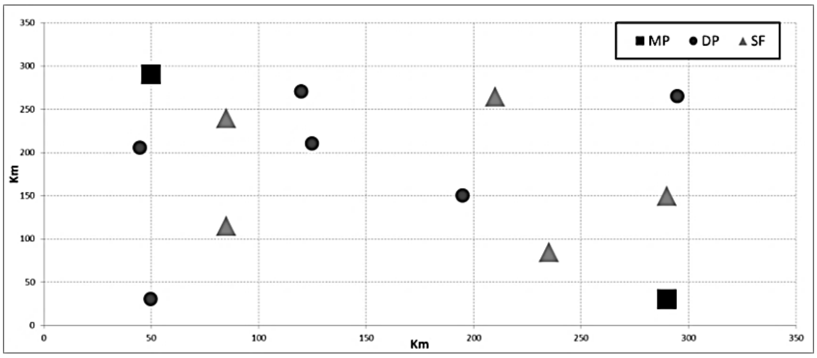

Fig. 2. Geographical distribution of *MPs*, *DPs* and potential *SFs* in the case study addressed.

Table 1 summarizes the optimal solutions found while their graphical representations are given in Fig. 3. The first two scenarios (1 and 2) consider similar demands at every *DP*, only modifying the unit waiting cost, which is assumed to be negligible in the second case. As expected, it is observed that the optimal solution is highly sensitive to the unit waiting costs. For the first scenario, the solution shows the convenience of installing two medium-size *SFs* supplying small *SFs*, one at every *DP*. The overall cost is minimized through expeditious demand fulfillment, due to the large unit waiting cost imposed. In contrast, the second scenario concentrates *PFs* in two medium-size *SFs*, taking advantage of the economies of scale in capital investment, material handling and transportation cost. Scenarios 3 and 4 heterogeneously distribute the overall demand, accounting for non-negligible waiting costs, yielding different network configurations, as shown in Fig. 3.

Table 1. Study cases results, costs components (MM USD) and formulation statistics.

Scenarios	***1***	***2***	***3***	***4***
Nº Large/Medium/Small SF	- / 2/ 6	- / 2 / -	1 / 1 / 4	0 / 2 / 4
NPC / TI	488 / 12.2	251.4 / 4.9	300 / 14.1	307 / 11.1
TPC_τ / TOC_τ	24.7 / 1.4	24.7 / 0.4	24.7 / 4.6	24.7 / 6.88
TTC_τ / TSC_τ	7.7 / 5.8	11.6 / 2.32	4.9 / 8.1	6 / 6.9
$Tffmc_\tau$ / $Tfmc_\tau$	0.6 / 2.8	0.1 / 1	0.5 / 2.5	0.5 / 2.2
TWC_τ	34.6	0	1.14	0.9
Equations	41,246	55,771	56,279	56,279
Positive/Integer Variables	7,239/912	9,467/1,148	10,028/1,148	10,028/1,148
CPU Time (s) /GAP	680 / 0	479 / 0	6,407 / 0	3,687 / 0

5 Conclusions

We have presented a novel approach for generalized supply chain design problems, taking into account distinctive characteristics of modern production and distribution networks. The novelties introduced in this work consist on: (1) avoiding the predetermination of echelons, allowing for free movements between storage facilities before demand fulfillment; and (2) capturing economies of scale governing the capital investment and operational costs. A MILP mathematical formulation is proposed, which permits to obtain efficient *SCN* designs with different types of facilities installed at different nodes, according to the relative importance of transportation, capital investment, operational and waiting costs. Results show interesting reactions against data changes, solving critical trade-offs along the supply chain structure. Future work will focus on the application of this model to larger case studies and industrial-size problems.

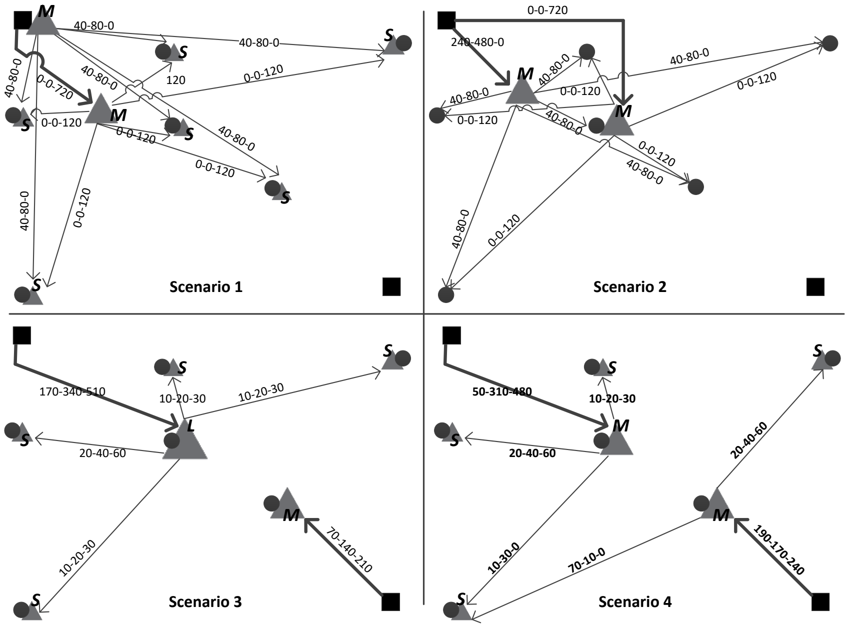

Fig. 3. Optimal *SCND* and *PF* flows for each scenario (*f1*/*f2*/*f3* in thousand tons per year).

References

1. International Monetary Fund, World Economic and Financial Survey (January 2017).
2. Shah, N.: Process industry supply chains: Advances and challenges. Computers and Chemical Engineering 29, 1225–1235 (2005)
3. Tsiakis, P., Shah,N., Pantelides, C.C.: Design of Multi-echelon Supply Chain Networks under Demand Uncertainty. Ind. Eng. Chem. Res. 40, 3585–3604 (2001)
4. Melo M.T., Nickel S., Saldanha-da-Gama F.: Facility location and supply chain management – A review. European Journal of Operational Research 196, 401–412 (2009)
5. Kalaitzidou M.A., LonginidisP.,Tsiakis P., Georgiadis, M.C.: Optimal Design of Multiechelon Supply Chain Networks with Generalized Production and Warehousing Nodes. Ind. Eng. Chem. Res. 53, 13125–13138(2014)
6. Laínez, G.K., Espuña A., Puigjaner L.: Flexible Design-Planning of Supply Chain Networks. AIChE Journal 55 1736–1753 (2009).
7. Farahani, R.Z., Rezapour S., Drezner T., Fallah S.: Competitive supply chain networkdesign: An overview of classifications, models, solution techniques and applications. Omega 45 92-118 (2014).